Evaluation of Uncertainty in Geometrical Measurements: A Pragmatic Approach

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ABSTRACT:

Modern manufacturing requires as a rule evaluation of conformance to geometrical specifications for products, and identification of sources of non-conformance if need be. As prescribed by such standard as ISO 9001, 9002, 9003, and particularly for geometrical product specifications (ISO 14253 1) the associated evaluation of measurement uncertainty is mandatory.

Conformance to geometrical specifications is currently evaluated using Coordinate Measuring Machines (CMM); however, at present, the main software installed on boards of CMM seldom caters for evaluation of the uncertainty of measurement according to the ISO Guide to the Expression of Uncertainty in Measurement (GUM). Acceptance of GUM at shop floor level is however far from universal, owing mainly to a cultural gap concerning treatment of uncertainty.

A case study is presented in this paper concerning measurement of circular geometrical features and evaluation of possible solutions for uncertainty estimation based on the propagation principle, according to GUM guidelines; a bootstrap method currently under evaluation, likely to be more appealing to less sophisticated users, is also mentioned. Results thus obtained are evaluated by comparison with the solution obtained through non-linear regression, for the case of actual measurements on cylindrical surfaces.

Keywords: CMM, Uncertainty, GUM.

1. INTRODUCTION

According to ISO 9000 series /1/ and other standards /2/, expression of uncertainty is mandatory for every test or measurement result. Easier said than done in a number of instances, e.g. when measurements are made with coordinate measuring machines (CMM) /3,4,5/, even relying upon the ISO Guide to the Expression of Uncertainty (GUM) /6,7/, which provides very effective ways for the expression of uncertainty. Uncertainty or maximum error pattern established in the course of machine calibration /8,9/ frequently produces a propagation of uncertainty so complex that the relevant estimate of uncertainty associated with the measurement result is seldom covered by the software of most CMM.

A major goal with uncertainty is straightforward derivation of an estimate thereof, consistent with the accuracy relevant to measurements performed according to current industrial practice. At industrial level tests may not be replicated liberally, owing to time and cost constraints, so that more than ten replications are seldom available; a reference threshold level for approximation may be established accordingly. Experience shows that simplified procedures may be arrived at leading to adequate approximation with no unduly complex mathematical procedures.

In order to devise a simple method for the evaluation of the uncertainty, and to check the adequacy of approximation of results obtained accordingly, a three-step approach was selected, consisting of:

- derivation of an advanced statistical evaluation to check the capability of simplified methods;

- development of a simplified method exploiting the advantages given by GUM;
- demonstration of an empirical method based on bootstrap techniques.

The methods proposed have been applied to the case of cylindrical surfaces.

2. ADVANCED STATISTICAL METHOD

The reference method adopted, based upon the maximum likelihood principle, was first developed for the case of a circular shape /10/. Extension to the case of a cylindrical shape may be derived provided that (as usual in the framework of CMM) measurement are performed in a local reference frame whose z axis is nearly coincident with the axis of the cylinder considered. In this case the shape may be adequately defined in terms of a series of circles corresponding to as many cylinder sections at different heights z. For each section j radius Rj and the relevant center coordinates α_j and β_j can be estimated individually. Center location and radius can provide indications on the evolution of surface along the z axis.

2.1. Statistical evaluation of the circular shape

Circular shape is characterized by an intrinsic non-linearity, thus representing a typical non-elementary problem. The solution is arrived at through an iterative procedure, hardly a problem in CMM measurement, since nominal values of center coordinates and radius are usually known. Each iteration estimates the matrix $\Delta = (\Delta R\ \Delta\alpha\ \Delta\beta)^T$ of increments $\Delta R\ \Delta\alpha\ \Delta\beta$ to be added to the previous evaluation R_0, of the radius, and α_0 and β_0, of the center coordinates, to obtain a best approximation. The method of the maximum likelihood yields as a solution

$$\Delta = [\mathbf{A}^T\mathbf{A}]^{-1}\mathbf{A}^T\mathbf{W} = \mathbf{C}\ \mathbf{W}$$

where **A** and **W** have the form given in Table 1.

Notice that matrix **A** is by and large defined by the experimental plan, and is nearly unaffected by the uncertainty of the experimental points; the same considerations apply for matrix **C**. On the other hand matrix **W** is strongly dependent upon measurement uncertainty. Accordingly an expeditious procedure based upon the method proposed by GUM may be devised. Using the law of uncertainty propagation in matrix form, the variance $\mathbf{u}^2(\Delta)$ may be evaluated from the variance $\mathbf{u}^2(\mathbf{W})$ of **W** as follows:

$$\mathbf{u}^2(\Delta) = \mathbf{C}\ \mathbf{u}^2(\mathbf{W})\ \mathbf{C}^T$$

Now the problem is to evaluate the variance $\mathbf{u}^2(\mathbf{W})$, and this can be done again using the suggestions of GUM. Direct statistical evaluation based upon analysis of residuals (deviations between measured and theoretical points) is not completely satisfactory, since it does not take into account possible systematic effects, and furthermore it is affected by geometrical irregularities corresponding to the measured sample. Therefore direct statistical evaluation based on the residuals can lead either to underestimation of the

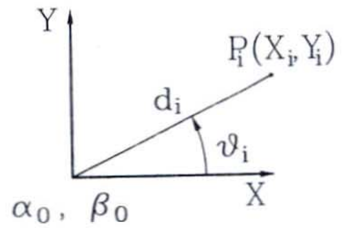

Fig. 1: Symbols used in Table 1

$$\mathbf{A} = \begin{bmatrix} 1 & \frac{X_1-\alpha_0}{d_1} & \frac{Y_1-\beta_0}{d_1} \\ . & . & . \\ 1 & \frac{X_i-\alpha_0}{d_i} & \frac{Y_i-\beta_0}{d_i} \\ . & . & . \\ 1 & \frac{X_n-\alpha_0}{d_n} & \frac{Y_n-\beta_0}{d_n} \end{bmatrix} = \begin{bmatrix} 1 & \cos\vartheta_1 & \sin\vartheta_1 \\ . & . & . \\ 1 & \cos\vartheta_i & \sin\vartheta_i \\ . & . & . \\ 1 & \cos\vartheta_n & \sin\vartheta_n \end{bmatrix} \qquad \mathbf{W} = \begin{bmatrix} d_1 - R_0 \\ . \\ d_i - R_0 \\ . \\ d_n - R_0 \end{bmatrix}$$

Table 1. Matrices **A** and **W**.

uncertainty, if bias due to the machine is neglected, or to overestimation, owing to undue systematic contribution of the measurand (e.g. lobing) not taken properly into account. In the latter case measurand irregularities may well remain hidden and inflate estimated measurement noise.

Residual systematic effects are evaluated during the calibration of the machine, and contained in the declaration of maximum error *E*. Therefore the approach indicated by the GUM, consisting in the propagation of the uncertainty starting from the uncertainty declared for the machine, comprehensive of general contribution as accuracy errors, hysteresis and others which may pass undetected a local or a complex repeatability test, can be relied upon to produce a better estimation of uncertainty. Such an approach is valid, of course, provided that the basic GUM hypothesis hold, namely that residual systematic effects and random effects have roughly the same range, and a number of concurrent causes of uncertainty (at least three) are present, as in the case considered.

The first step deals with maximum error *E* found during the calibration /8/ and the relevant variances.

Variances $u^2(X)$, $u^2(Y)$ and $u^2(Z)$ of coordinates *X*, *Y* and *Z* given by the machine may be evaluated with the method of type B uncertainty contributions:

$$u^2(X) \approx u^2(Y) \approx u^2(Z) \approx \frac{E^2}{3}$$

The second step consists in propagating these variances to the variances of the elements of matrix **W**, which take the form:

$$W_i = \sqrt{(X_i - \alpha)^2 + (X_i - \beta)^2} - R$$

Propagation can therefore be performed using the function:

$$w = \sqrt{(x-\alpha)^2 + (y-\beta)^2} - r$$

and finally:

$$u^2(W) \cong \left(\left. \frac{\partial w}{\partial x} \right|_{P_i} \right)^2 u^2(X) + \left(\left. \frac{\partial w}{\partial y} \right|_{P_i} \right)^2 u^2(Y) = \\ = \left(\frac{X_i - \alpha}{W_i + R} \right)^2 \frac{E^2}{3} + \left(\frac{Y_i - \beta}{W_i + R} \right)^2 \frac{E^2}{3} = \frac{E^2}{3}$$

Accepting for the sake of expediency the hypothesis of absence of covariances, as suggested by GUM at a first step approximation unless there is evidence to the contrary, the variance-covariance matrix for Δ is given by:

$$\mathbf{u}^2(\Delta) = \mathbf{C}\,\mathbf{C}^T\,E^2/3$$

Geometrical parameters for a circular shape and related uncertainties are thus determined.

2.2. Advanced statistical evaluation of cylindrical shape features

As stated above, the parameters of a (circular) cylinder can be estimated by measuring a number of circular shapes and determining the relevant radii Rj and the center coordinates α_j and β_j. Cylinder radius R_c may be taken as the average radius of the q sections examined, and the axis may be approximated by the straight line fitted with the least squares method:

$$R_c = \frac{\sum_{j=1}^{q} R_j}{q} \qquad \begin{cases} \alpha = A_0 + A_1 z \\ \beta = B_0 + B_1 z \end{cases}$$

Again, the uncertainty of the parameters thus determined, namely R_c, A_0, A_1, B_0 and B_1, can be evaluated according to the rule of uncertainty propagation.

The case of the radius is straightforward, since:

$$u^2(R_c) = \frac{\sum_{j=1}^{q} u^2(R_j)}{q^2}$$

Parameters A_0, A_1, B_0 and B_1 may be dealt with similarly. Derivation of matrix **A** of coefficients A_0 and A_1 in terms of matrix **Z** of the values Zj of the circular sections and matrix α of coordinates α_j of the relevant centers takes the form:

$$\mathbf{A} = [\mathbf{Z}^T\mathbf{Z}]^{-1}\mathbf{Z}^T\,\alpha = \mathrm{M}\,\alpha$$

Therefore, taking again $\mathbf{M} = [\mathbf{Z}^T\mathbf{Z}]^{-1}\mathbf{Z}^T$ as nearly independent from measurement uncertainty (affecting mainly α) we can write:

$$\mathbf{u}^2(\mathbf{a}) = \mathbf{M}\,\mathbf{u}^2(\alpha)\,\mathbf{M}^T$$
$$\mathbf{u}^2(\mathbf{b}) = \mathbf{M}\,\mathbf{u}^2(\beta)\,\mathbf{M}^T$$

thus defining the uncertainty in the determination of the parameters of the cylindrical shape. This defines a reference for evaluating a pragmatic method, based on uncertainty propagation assessed on a deterministic approach.

3. PRAGMATIC METHOD PROPOSED

Under working conditions consistent with GUM guidelines, a pragmatic approach may be adopted in order to assess the uncertainty affecting determination of parameters of a cylinder. The proposed method estimates the parameters via a system of deterministic equations based on five measurement points P_j. Standard uncertainty assessment is evaluated again through variance propagation from the maximum error E.

Taking into account CMM characteristics, the axis of the cylinder examined may be considered as nearly coincident with the Z-axis of a local reference system. Accordingly, the parameters to be determined are radius r, axis inclinations projected on the plane x-z (δ) and on the plane y-z (η), and coordinates a_0 and b_0 of intersection of the cylinder's axis with plane x-y, as shown in Fig. 2.

Taking into account points $P(x_p, y_p, z_p)$ and $P_i(x_i, y_i, z_i)$, we may write

$$\begin{aligned} r^2 &= (x_i - x_p)^2 + (y_i - y_p)^2 + (z_i - z_p)^2 = \\ &\approx (x_i - a_0 - z_i\delta)^2 + (y_i - b_0 - z_i\eta)^2 = \\ &\approx (x_i^2 + y_i^2) - 2x_ia_0 - 2x_iz_i\delta - 2y_ib_0 - 2y_iz_i\eta \end{aligned}$$

That is, given five measured points, a system of five equations with five unknown is obtained as:

$$\begin{cases} (x_1^2 + y_1^2) - 2x_1x_0 - 2x_1z_1\delta - 2y_1y_0 - 2y_1z_1\eta \approx r^2 \\ (x_2^2 + y_2^2) - 2x_2x_0 - 2x_2z_2\delta - 2y_2y_0 - 2y_2z_2\eta \approx r^2 \\ (x_3^2 + y_3^2) - 2x_3x_0 - 2x_3z_3\delta - 2y_3y_0 - 2y_3z_3\eta \approx r^2 \\ (x_4^2 + y_4^2) - 2x_4x_0 - 2x_4z_4\delta - 2y_4y_0 - 2y_4z_4\eta \approx r^2 \\ (x_5^2 + y_5^2) - 2x_5x_0 - 2x_5z_5\delta - 2y_5y_0 - 2y_5z_5\eta \approx r^2 \end{cases}$$

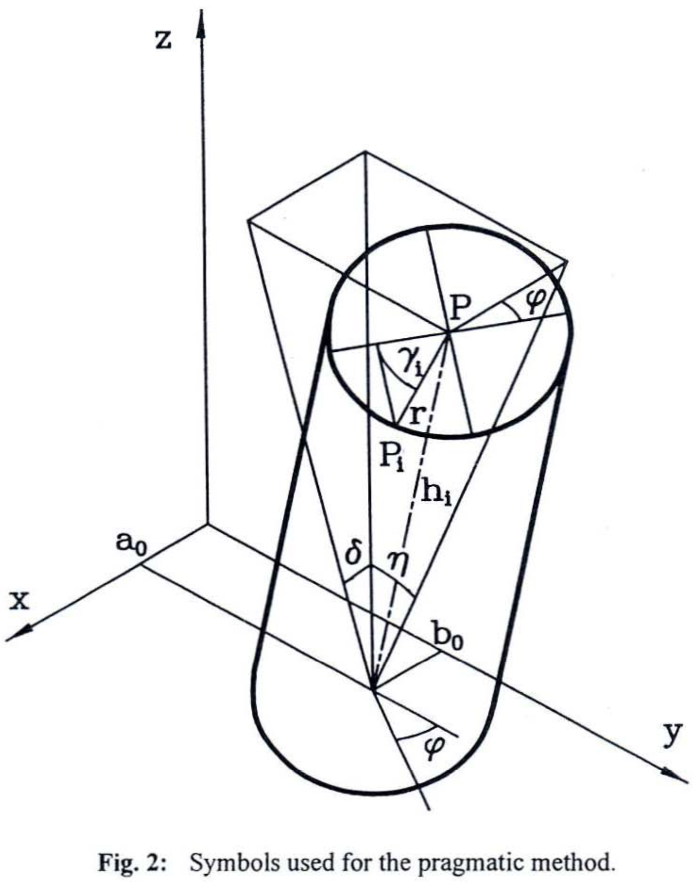

Fig. 2: Symbols used for the pragmatic method.

Table 2
Uncertainty budget for parameter R

Independent variables ξ_i	Variability field $\pm a(\xi_i)$	$u^2(\xi_i)$	$c_i = \left(\frac{\partial R}{\partial \xi_i}\right)$	$u_i^2(R)$
X_1	E	$E^2/3$	c_{x1}	$c_{x1}^2 E^2/3$
...	...	...	...	...
X_5	E	$E^2/3$	c_{x5}	$c_{x5}^2 E^2/3$
Y_1	E	$E^2/3$	c_{y1}	$c_{y1}^2 E^2/3$
...	...	...	...	...
Y_5	E	$E^2/3$	c_{y5}	$c_{y5}^2 E^2/3$
Z_1	E	$E^2/3$	c_{z1}	$c_{z1}^2 E^2/3$
...	...	...	...	...
Z_5	E	$E^2/3$	c_{z5}	$c_{z5}^2 E^2/3$
		Variance $u_c^2(R)$		$\sum u_i^2(R)$
		Standard uncertainty		$u_c(R)$
		Coverage factor k		2
		Expanded uncertainty		$k \cdot u_c(R)$

Such a deterministic system is not used to estimate the cylinder's parameters; however, it may be exploited for expressing the uncertainty. The usual method of uncertainty propagation can be organized in the form represented in Table 2 for parameter R taken as a dependent variable.

Tables for the remaining parameters are quite similar, but for the case pertaining to sensitivity coefficients c_i whose estimation is more involved, owing to the form of partial derivative of the extrinsic equations. Partial derivatives are to be computed substituting variables ξ_i, that is:

$$\left.\frac{\partial R}{\partial \xi_i}\right|_{P_i} \approx \left.\frac{R(\xi_i + \delta\xi_i) - R(\xi_i)}{\delta\xi_i}\right|_{P_i}$$

Evaluation of the standard uncertainty of parameter R, and in the same way of all of the cylinders' parameters, is thus made possible.

Discussion of two main points is in order, namely:

- the uncertainty is definitely affected by the experimental plan, therefore both advanced and pragmatic methods are sensitive to the relevant experimental plan. The problem for the pragmatic method boils down to selecting five measurement points properly representing the set of points measured with the CMM in actual practice, a set always in excess of five.
- the combined uncertainty is sensitive to the number n of measurement points, as the amount of information increases with n. Should points be

evenly spaced on the accessible portion of the cylinder's surface, the effect of their number might be deemed similar to that of the degrees of freedom in affecting the standard deviation of sample mean. A correction for the number n of points used with the CMM against the 5 points used for the deterministic evaluation of the uncertainty can be given by the ratio $\sqrt{5}/\sqrt{n}$.

NUMERICAL SIMULATION METHOD

Evaluation of uncertainty – a core subject in both statistical inference and metrology – lends itself readily to a numerical approach, as made attractive by the evolution in information processing. The jackknife technique draws from the available data set of size n as many different samples of size n-1, by leaving out in turn one observation at a time. Bias and standard deviation of the statistics considered are estimated from the set of samples thus obtained /11,12,13/.

On the other hand, an unlimited number of bootstrap samples of size n may be obtained by drawing with replacement as many items from the available data set of size n too. Actually the process is repeated say from 100 up to 400 times or so, and inferences on the distribution of the statistics considered are derived from the set of samples thus formed. Bootstraps and jackknives are obviously closely related, and performances are comparable in the case of linear functions; in the non-linear case the bootstrap is found to outperform the jackknife, as the latter may fail in the case of non-smooth, non-differentiable functions, see for instance /14,15,16/.

The capability of extracting inferences in a straightforward, user friendly way, without overtaxing the analytical skills of users who may not necessarily have a professional statistician's background, is one of the more attractive features of the bootstrap. Engineers and metrologists may therefore exploit in full the opportunities offered by GUM, and concentrate on understanding the real meaning of the data set at hand instead of wrestling with involved analytical derivations.

In the case currently being evaluated, estimates of coordinates of center and measure of radius were obtained from a set of measurements with a CMM performed on a (nominally) cylindrical artifact. Empirical distributions of estimates exhibit features definitely comparable with those of uncertainty estimates obtained according to standard propagation of variance procedure as stated in GUM.

Extension of the method developed for parameter estimation and uncertainty assessment for a circular shape /10/ to more complex features, namely cylindrical surfaces, may be approached as a stepwise process. Two kinds of errors only are considered as a first step for the axis, namely departure from perpendicularity to horizontal (datum) plane, and departure from linearity as approximated by curvature in a plane. A broader range of geometrical errors may be taken into account at a later stage, should lack of fit pattern indicate substantial inadequacy of the simple error model proposed. Given k sets of measurements performed approximately on (evenly) spaced horizontal planes, departure of axis from nominal (vertical) direction may be estimated if $k \geq 2$, and curvature if $k \geq 3$. More complex models for the axis, e.g., spatial shapes, may be required for the case of long, slender shafts or small, deep bores, and dealt with as extensions of the case at hand. And the more complex the shape the more apparent are the advantages offered by data based simulation, as no unduly complex analytical manipulations are thereby entailed.

5. MEASUREMENTS AND DISCUSSION

The pragmatic method adopted has been tested experimentally by measuring cylindrical surfaces, different in the ratio diameter to length, and also on some parts thereof, typically when a part only of the cylindrical surface is accessible or even does not exist, for instance when a tile-shaped part or a fillet is considered. The CMM measurement results presented in Table 3 were obtained on a whole cylinder (Case 1) and on a limited part of it, that is a sixth (Case 2) and a twelfth part thereof (Case 3). Uncertainty evaluations derived with both advanced and pragmatic methods are also shown.

Some considerations are in order. As expected, uncertainty increases for decreasing values of aperture angle γ defining the surface considered, with some remarkable distinctions. For angle δ the loss is but

Table 3

Comparison of the measurement results and of the standard uncertainty evaluation (equalized for nine measurement points), obtained with the CMM, the advanced and the pragmatic method

Case 1	Cylinder, aperture angle γ = 360°			
Parameter	CMM	Advanced		Pragmatic
	Value	Value	St. Unc.	St. Unc.
r/mm	16,006	16,005	$8{,}2 \cdot 10^{-4}$	$1{,}8 \cdot 10^{-3}$
a_0/mm	0,003	-0,003	$3{,}0 \cdot 10^{-3}$	$5{,}2 \cdot 10^{-3}$
b_0/mm	0,000	0,002	$3{,}1 \cdot 10^{-3}$	$3{,}3 \cdot 10^{-3}$
δ/rad	-0,0002	-0,0001	$2{,}3 \cdot 10^{-4}$	$4{,}1 \cdot 10^{-4}$
η/rad	-0,0002	-0,0004	$2{,}4 \cdot 10^{-4}$	$2{,}8 \cdot 10^{-4}$

Case 2	Fillet, aperture angle γ = 60°			
Parameter	CMM	Advanced		Pragmatic
	Value	Value	St. Unc.	St. Unc.
r/mm	15,992	15,974	$1{,}2 \cdot 10^{-2}$	$1{,}9 \cdot 10^{-2}$
a_0/mm	0,016	0,022	$3{,}5 \cdot 10^{-2}$	$2{,}1 \cdot 10^{-2}$
b_0/mm	0,015	0,021	$5{,}4 \cdot 10^{-3}$	$5{,}6 \cdot 10^{-3}$
δ/rad	-0.0001	-0,0007	$4{,}2 \cdot 10^{-4}$	$2{,}4 \cdot 10^{-4}$
η/rad	-0.0001	0,0000	$2{,}7 \cdot 10^{-3}$	$4{,}2 \cdot 10^{-4}$

Case 3	Fillet, aperture angle γ = 30°			
Parameter	CMM	Advanced		Pragmatic
	Value	Value	St. Unc.	St. Unc.
r/mm	16,040	16,064	$5{,}0 \cdot 10^{-2}$	$8{,}9 \cdot 10^{-2}$
a_0/mm	-0,033	0,074	$1{,}4 \cdot 10^{-1}$	$9{,}2 \cdot 10^{-2}$
b_0/mm	0,002	0,018	$1{,}0 \cdot 10^{-2}$	$2{,}8 \cdot 10^{-2}$
δ/rad	-0,0002	0,011	$8{,}0 \cdot 10^{-4}$	$7{,}4 \cdot 10^{-4}$
η/rad	0,0004	0,002	$1{,}0 \cdot 10^{-2}$	$2{,}4 \cdot 10^{-3}$

marginal, as explained taking into account the position of a fillet having its symmetry plane parallel to plane x-z. Taking aperture angle $\gamma = 0$ the fillet degenerates into a straight line, the estimation of whose slope entails no problem. Accordingly, x and z coordinates are always well determined and so is angle δ. On the other hand, the smaller is aperture angle γ, the worse becomes estimation of angle η, a fact duly reflected by the results obtained.

By and large, estimates obtained with different approaches exhibit a reasonable agreement, taking into account the rather large uncertainty affecting estimates of uncertainty in the case at hand. With four degrees of freedom only, as when measuring a cylindrical surface with but nine probed points (consistently with industrial practice), the relevant coefficient of variation reaches the 85% level.

The deterioration in precision for Cases 2 and 3 as compared to Case 1 indicates that the uncertainty evaluation should be done with direct reference to the real shape and dimensions of the surface examined; therefore, provision for evaluation of the uncertainty on the boards of CMM is highly desirable.

The pragmatic method proposed is simple and straightforward; however, the inherent weakness connected with using just a handful of points should not be underestimated.

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